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Hieronimus Cardanus 57

THE MATHEMATICS TEACHER

Volume XXIII No. 7

Edited by William David Reeve

Building Tests in Junior High School Mathematics

By C. N. Stokes
University of Minnesota

The fundamental problem in teaching junior high school mathematics resolves itself into a matter of effecting real learning. The pupil must have mastery of the minimum essentials. He must have those skills and understandings which are deemed the necessary equipment of every normal individual who must make the necessary adaptations to the environmental conditions of the well-ordered life. He must have the ability which stands the test of normal and habitual use under practical conditions in the everyday life situations. Training in junior high school mathematics must add materially to his stock of abilities which are needed in making adjustments to the demands of society.

One consideration in the process through which the learning may be acquired is to determine the effectiveness of instruction. A diagnosis must be made periodically in order that any deficiencies in learning may be located and corrected. It is this diagnosing that concerns us in this discussion. The aim is to give the reader a practical technique which the writer has been using for several years in his mathematics courses in the University of Minnesota High School.

This discussion is really a sequel to an article appearing in the

November, 1928, issue of THE MATHEMATICS TEACHER entitled, Educational Tests—to Standardize or not to Standardize.[1]

In this article Professor Reeve gives us his views on the status of educational tests in secondary school mathematics. We are shown the short-comings of the standardized test and the values of the new-type test. We are shown that the modern trend is a movement away from the commericial standardizations toward the new-type examinations which more nearly fit the materials as they are presented in the respective school organizations. Professor Reeve states that:

"Teachers not only can learn how to make objective tests that will have both measuring and diagnostic value, but they can also learn to use them intelligently. This ability to use the tests will increase in proportion to the progress that teachers make in understanding more scientific methods of measurement."

The criteria set up for the construction of tests are:

"1. Every test should attempt to measure a pupil's ability to master the subject matter that has been presented to him. This means that the one who makes the test must discern clearly the objectives in the topic or course and must build the test so as to measure the extent to which these objectives have been realized.

"2. Every test should emphasize primarily those parts of the subject matter which are fundamental and to which the pupils have directed the most attention. In other words, the test must be comprehensive. Nothing should receive attention that is not worth perpetuating in the course. If these two points are secured, we may say that the tests are valid.

"3. The scoring of each test should be so arranged that in scoring them all teachers may obtain exactly the same results, or the same teacher may on giving a test a second time obtain the same result that he got at first. The test should be so constructed that it is self administering. In other words, the tests must be objective.

"4. Every test should be reliable, that is, it should measure what it measures to a satisfactory degree of accuracy.

"5. Every test should be so constructed that it is possible to set some form of standard of achievement for a pupil."

In the construction of our tests a very definite technique is followed. The first step is an analysis of the course to identify the specific

[1] Reeve, W. D., "Educational Tests—to Standardize or not to Standardize," *The Mathematics Teacher*, Vol. XXI, November, 1928, pp. 369-90.

abilities which constitute the minimum essentials to be emphasized in the teaching and which must be measured by the test. To illustrate this point, the unit on "adding algebraic expressions" contains upwards of 75 different learning processes. This number may be increased or diminished when we know more about methods of learning.

The second step is to draft a set of questions which are objective, comprehensive, reliable, and economical as to administration and scoring. The following is a descriptive account of the procedure: (1) Administer this first draft to a small number of pupils who, through performance, have demonstrated that there has come a real understanding of content. These results are then tabulated as shown in the following table:

RESPONSES MADE BY TEN PUPILS ON THE FIRST DRAFT OF UNIT TEST ON FORMULAS

Pupils	Items on Test															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
I........	x	x	x	0	x	x	x	x	x	x	0	x	x	x	x	x
II........	x	x	x	x	0	x	x	x	0	0	x	x	x	x	x	x
III........	0	x	x	x	x	x	x	x	x	0	x	x	x	x	0	x
IV........	x	0	x	x	x	0	x	x	x	0	x	x	x	x	x	x
V........	x	x	x	x	x	x	x	x	x	0	x	x	0	x	x	x
VI........	x	x	0	x	x	x	x	x	x	x	0	0	x	x	x	x
VII........	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	0
VIII........	x	x	x	x	x	x	x	x	x	0	x	x	x	x	x	x
IX........	x	x	x	x	x	x	x	x	x	0	x	x	x	x	x	x
X........	x	x	x	x	x	x	x	x	x	0	x	x	x	x	x	x
Total rights	9	9	9	9	9	9	10	10	9	3	8	9	9	10	9	9

This is a hypothetical case but it illustrates the point that a careful examination of such a picture will locate any mistakes. Item numbered 10 is an illustration. The defect must be corrected. Several errors may have been prevalent when this item was included in the test. It may have been clouded emphasis in teaching, misjudged importance as an integral part of the test, faulty language in which it was couched, etc. Whatever the cause may have been, the item is placed aside and a substitution is made. (2) Administer the test a second time, but in this case, to the slowest pupils of the group. They should not do nearly so well as the first group, yet, any misfires can be located so as to facilitate further corrections. (3) Administer the test a third time to the remaining pupils who are taking

the subject (70 pupils constitute a satisfactory group). These results are scrutinized and the test is laid aside until a second group is available. The procedure is then repeated for further improvement upon the test. The following notes which were taken from our files will illustrate the statistical treatment which follows the third administration:

General Mathematics—Ninth Grade
Test—Mastery Test—Unit of Formulas
(Due to limited space, the test and the distribution of scores are omitted)

Description of test:

Three parts:

Part I. 50 yes-no questions, hereafter referred to as 50 yn
Part II. 20 alternative choice questions, hereafter referred to as 20ac
Part III. 10 problems, hereafter referred to as 10p

Reliabilities:

Entire test 86
Part I 76
Part II 66
Part III 67

Required length:

For purposes of individual diognosis and placement, a reliability of 95 is desirable.

Setting 95 as the required reliability, and assuming that each of the three parts of the test are to be made equally reliable, so that equal weight may be given them, each part should have a reliability of 86 which is the present reliability of the entire test.

To give each part a reliability of 86, the following lengths would be necessary for each part.

Part I, 50 yn.........................increase by 40 items—total 90
Part II, 20ac.........................increase by 43 items—total 63
Part III, 10p.........................increase by 22 items—total 32

Options:

1. Be satisfied with lower reliability.
2. Increase to required length.
3. Increase the reliability without increasing the length of test. This requires an inspection of individual items, throwing out the undesirable and replacing them with desirable ones.

Recommendations:

1. Lengthen the last two parts of the test so that the reliability of each is equal to that of the first.

This requires lengths as follows:

Part II, 20ac..............................increase 10 items—total 30
Part III, 10p..............................increase 5 items—total 15

This lengthening will give each part a reliability of 76 and the reliability of the total test as 90.

2. Apply option 3 to bring the reliability of the lengthened test up to requirement.

   This procedure prevents the cumbersomeness of administration and scoring which would exist if option 2 be applied.

For an explanation of this statistical treatment, the reader is referred to Dr. Truman L. Kelley's treatise, "Statistical Methods."[1] Another reference, which is likely to be more usable, is a table worked out by Edgerton and Toops.[2] It concerns option two above, primarily.

It is to be noted that if this test is to be used in successive years, it will be advisable to consider further the individual items, rejecting those which cause low reliability and substituting others for trial in a new form. In time a very suitable test can be constructed, the reliability and validity of which can be little open to question on account of the nature of the items.

Every school should build its own tests. No standardized test will measure the minimum essentials as taught in most school systems. We may be a little more fortunate than some, for, on some of our tests, an appreciable amount of assistance has come from outside schools. The writer offers two courses in special methods during the summer. The people attending these classes go back to their respective positions and present the courses as they were outlined and emphasized in their summer work. The tests are administered and the results sent back to our department. This assistance gives us a much greater number of pupil responses and thus aids in determining the reliability of the tests. There is no need of anyone thinking, however, that it takes a great number of pupils to make valuable improvement upon the tests administered in the school.

To illustrate the product of our efforts, we present below one of the forms of the test on the unit of ninth grade demonstrative geometry. It has a reliability of 93.

Mastery Test Name........................

Unit—Demonstrative Geometry—Ninth Grade........U. H. S.

Part I—yes or no responses

(Place response in front of question)

1. Must a parallelogram be a quadrilateral?
2. Are the acute angles of a right riangle always complementary?

[1] Kelley, Truman L., "Statistical Method." New York: The Macmillan Company, 1924, pp. 205-206.

[2] Edgerton, H. A., and Toops, H. A., "A Table for Predicting the Validity and Reliability Coefficients of a Test when Lengthened," *The Journal of Education Research,* Vol. 18, October, 1928, pp. 225-34.

3. Is the sum of the interior angles of a triangle always equal to 360 degrees?
4. Does every triangle have one obtuse angle?
5. Are the opposite angles of a parallelogram always equal?
6. Are two triangles always congruent if they have three sides of one respectively equal to three sides of the other?
7. Is a quadrilateral a parallelogram if two of its sides are equal and parallel?
8. If two angles of a triangle are equal, is the triangle always equilateral?
9. Are diameters of circles ever used as chords?
10. Is an equilateral triangle always isosceles?
11. Is a trapezoid a parallelogram?
12. Are the diagonals of a parallelogram always equal?
13. Can an angle ever be greater than its supplement?
14. May a right triangle be isosceles?
15. Can more than one straight line be drawn from a given point?
16. Must a trapezoid have two equal sides?
17. Is the altitude of a triangle ever drawn to the mid-point of the base?
18. Are two triangles always congruent if three angles are respectively equal?
19. Is the complement of an angle of 96 degrees less than its supplement?
20. Must the diagonals of a quadrilateral always bisect each other?
21. Does a triangle always have more than one acute angle?
22. Does an interior angle of a triangle ever equal an exterior angle?
23. If two straight lines intersect, are adjacent angles always supplementary?
24. If two straight lines intersect, making the adjacent angles equal, are the lines always perpendicular?
25. If two right triangles have a hypotenuse and a side of one equal respectively to the same of the other, are they necessarily congruent?
26. Is the sum of all the angles about a point in a plane always equal to $180°$?
27. If two angles of one triangle are equal respectively to two angles of another, are the third angles always equal?
28. Are all the angles of an isosceles triangle ever equal?
29. When one straight line is drawn meeting another, are the adjacent angles thus formed always supplementary?
30. Can two perpendiculars be drawn from an outside point to a line?
31. Are the alternate interior angles always equal when two lines are cut by a transversal?
32. If two triangles have the same size, does it mean that they are always congruent?
33. Is a rectangle classed as a parallelogram?
34. If the sum of two adjacent angles is $180°$, do their exterior sides form a straight line?
35. If two triangles have two angles and a side of one equal to two angles and a corresponding side of the other, are the triangles always congruent?

Part II—Completion

(There are three divisions (i, ii, iii) in this part of the test. Follow the directions as given under each.)

(i) Define the following terms:

1. Adjacent angles.
2. Acute angle.
3. Complementary angles.
4. Vertical angles.
5. Congruent figures.
6. Converse of a theorem.
7. Corresponding parts.
8. Octagon.
9. Quadrilateral.
10. Isosceles triangle.
11. Altitude of a triangle.
12. Hypotenuse.
13. Median of a triangle.
14. Parallelogram.
15. Regular polygon.
16. Supplementary angles.
17. Theorem.
18. Axiom.
19. Parallel lines.
20. Diagonal of a figure.

(ii) Fill in the following outline:

1. Two triangles are congruent if
   a.
   b.
   c.
   d.
2. Two right triangles are congruent if
   a.
   b.
3. If two parallel lines are cut by a transversal
   a.
   b.
   c.
4. Two lines are parallel if they are cut by a transversal so that
   a.
   b.
   c.
5. Two angles are equal if they are
   a.
   b.
   c.
   d.

6. A parallelogram has
   a.
   b.
   c.
   d.
   e.
7. A quadrilateral is a parallelogram if
   a.
   b.
   c.
   d.
   e.

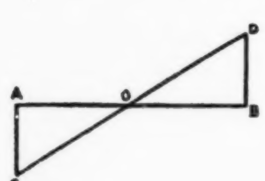
(iii) Finish the following statements:

1. The sum of the three interior angles of a triangle is
2. The sum of the three exterior angles of a triangle is
3. The sum of the interior angles of a polygon of n sides is
4. The sum of the exterior angles of a polygon of n sides is
5. An exterior angle of a triangle is equal to
6. If a triangle has two equal angles,
7. A diagonal divides a parallelogram into
8. If three or more parallels intercept equal segments,
9. If the arms of one angle are respectively perpendicular to the arms of another,
10. Two lines parallel to a third line,
11. The two acute angles of a right triangle are
12. If a line is parallel to one side of a triangle and bisects another side,
13. Two lines perpendicular to the same line are
14. Through a given point outside a given line,

Part III

(Complete the unfinished statements)

(i) Drawing Conclusions

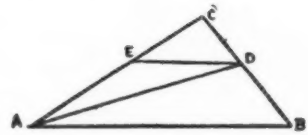


1. In the accompanying figure, if we have
   Given: AC equal to BD,
   AC and BD perpendicular to AB,
   We can prove that
   1 ....
   2 ....
   3 ....

2. In the accompanying figure, if we have

Given: Triangle ABC
AD bisecting angle CAB
DE parallel to AB

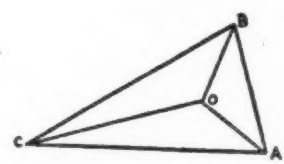
We can prove that
1 ....
2 ....
3 ....



3. In the accompanying figure, if we have

Given: Triangle ABC with AC equal to BC
AO bisects angle BAC
BO bisects angle ABC
CO is drawn

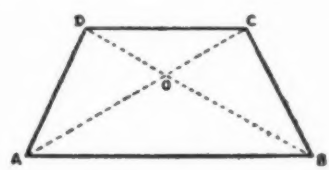
We can prove that
1 ....
2 ....
3 ....
4 ....



4. In the accompanying figure, if we have

Given: Isosceles trapezoid ABCD
AD equals DC equals CB
Diagonals AC and BD

We can prove that
1 ....
2 ....
3 ....
4 ....



(ii) Constructions

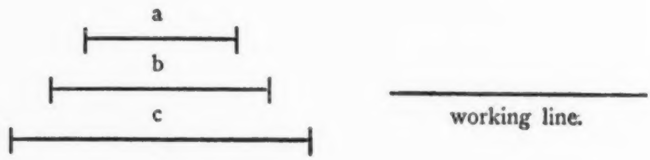
1. Construct a rectangle whose length is 4 cm. and whose width is 3 cm.

working line

2. Construct a square whose side is 2 cm. long.

working line

3. Construct a triangle, using the three sides a, b, and c, given below.



4. Construct a triangle using the two sides and the included angle B given below.

5. Construct a triangle, using the two angles and the included side given below.

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Eleventh Year Mathematics Outline

By W. D. Reeve and his Students
Teachers College, Columbia University
New York, N. Y.

The following outline is based on the assumption that the function concept is the most important idea to be stressed throughout the eleventh year. Instead of beginning the course with the traditional formal review of the ninth year mathematics, the function idea is introduced at once through variation and formulas. Without doubt the fundamental principles of algebra will need to be reviewed, but only as needed in connection with a study of the topics listed in the outline.

There is no assurance that the use of such an outline as this will result in anything other than traditional teaching. Much depends on the way the outline is used. If the function concept is to be developed there must be something more than mere evaluation of formulas, making graphs, and solving equations. The dependence of variables and the way things change together will need to be continually pointed out and discussed.

General References

1. Breslich, E. R. Developing Functional Thinking in Secondary School Mathematics. Third Yearbook of the National Council of Teachers of Mathematics, Bureau of Publications, Teachers College, New York.
2. Everett, J. P. The Fundamental Skills in Algebra. Bureau of Publications, Teachers College, New York.
3. Fourth Yearbook of the National Council of Teachers of Mathemathics, Bureau of Publications, Teachers College, New York.
4. Nunn, The Teaching of Algebra, Longmans.
5. Nunn, Exercises in Algebra, Books I and II, Longmans.
6. Smith, D. E. History of Mathematics, Ginn, Vols. I and II.
7. Smith, D. E. The Progress of Algebra, Ginn.
8. Thorndike, The Psychology of Algebra, Macmillan.

I. *Variation (Including an informal treatment of Ratio and Proportion)*

A. Objectives:

1. To stress the function concept.
2. To present algebraically and graphically direct variation, (uniform and non-uniform) inverse variation, and joint variation.
3. To develop an understanding of a ratio as an operation in division, and of a proportion as the equality of two or more ratios.
4. To emphasize the fact that a proportion is an equation and to use the principles of fractions and equations in teaching the changes that can be made in the form of the proportion without destroying the equality.
5. To develop the ability to analyze formulas from the standpoint of their description of variation.
6. To develop a working concept of the following expressions: function, dependent variable, independent variable, constant, varies directly as, varies inversely as, varies jointly as, uniform direct variation, inverse variation, joint variation, ratio, proportion, directly proportional to, and inversely proportional to.

B. References:

1. Longley and Marsh, Algebra, Bk. II, pp. 86–107; 167–185.
2. Nunn, The Teaching of Algebra, pp. 109–110, 117–120; 145–155.
3. Nunn, Exercises in Algebra, Part I, pp. 103–106; 126–134.
4. Nyberg, Second Course in Algebra, pp. 89–102.
5. Schorling, Clark and Rugg, Modern Mathematics.—Briefer Course, pp. 337–368.
6. Smith and Reeve, Essentials of Algebra, Complete Course, pp. 257–272.
7. Smith and Reeve, Essential. of Algebra, Bk. II, pp. 1–22; 35–36.
8. Swenson, High School Mathematics, pp. 87–93; 281–299.
9. Sykes and Comstock—A Second Course in Algebra pp. 36–37.

C. Subject Matter:

1. Function concept as related to variation:
   (See Schorling, Clark and Rugg, pp. (337–341) and Smith and Reeve—Book II—pp. (1–22).
   a. Numbers that change together. (Let the class give illustrations. Example—the postage on a parcel changes with the distance).
   b. Definition of function. (Use the illustrations from the preceding discussion as basic material).
   c. The use of the formula to express functional relation.
      (1) Dependent and independent variable, and constant.
   d. Functional notation.
2. Direct Variation.
   (See Schorling, Clark and Rugg—pp. 342–356) and Smith and Reeve—Complete Course, pp. 260–266).
   a. Uniform:
      (1) Meaning—illustrations given by the class.
      (2) Development of the formula and graph.
      ($y=kx$, preferred form)
      (3) Practice in various ways of writing formulas expressing direct variation.

$p=4s \qquad p=ks$

$\frac{p}{s}=4 \qquad \frac{p}{s}=k$

Since k represents any other $\frac{p}{s}$, the following form results:

$\frac{p_1}{s_1}=\frac{p_2}{s_2}$

      (4) Ratio as a quotient

$\frac{C}{d}=\pi$

      (5) Proportion as the equality of 2 ratios.
   b. Direct variation which is not uniform
      (1) Meaning

(2) Discussion of formulas and graphs depicting non-uniform direct variation and contrasted with those for uniform direct variation.

(3) Problems

(In graphing formulas containing π, let π be the unit of measure along one axis.)

3. Inverse variation.

(See Schorling, Clark, and Rugg—pp. 357–362.

a. Meaning

Good illustration of inverse variation—Two towns are 200 miles apart. How does the time required in going from one to the other change with the rate? What is constant?)

b. Development of the formula and graph.

($x=\dfrac{k}{y}$, preferred form.)

c. Practice in various ways of writing formulas expressing inverse variation.

$rt=200$ $\qquad\qquad r=\dfrac{k}{t}$

$rt=k$ $\qquad\qquad t=\dfrac{k}{r}$

Since k (in the second formula) may represent any other rt, $r_1t_1=r_2t_2$, and using the laws of proportion

$$\frac{r_1}{r_2}=\frac{t_2}{t_1}$$

d. Comparison of direct and inverse forms:

Direct	Inverse
$y=kx$	$y=k/x$
. . . .	. . . .
$\dfrac{C_1}{C_2}=\dfrac{r_1}{r_2}$	$\dfrac{P_1}{P_2}=\dfrac{V_2}{V_1}$

4. Joint Variation

(See Schorling, Clark, and Rugg—pp. 363–364.)

a. Definition of joint variation—introduced by a dis-

cussion of formulas in which more than two variables change together. Example $I = prt$.

b. Problems

II. *Formulas*

A. Objectives:

1. To show the need for understanding the language of formulas.
2. To develop the ability to make and interpret formulas and their graphs.
3. To develop the ability to solve formulas for any letter. Nunn calls this "changing the subject of a formula."

B. References:

1. Nunn—Teaching of Algebra, pp. 63–67.
2. Nunn—Exercises in Algebra—Bk. I., pp. 1–32.
3. Mullins and Smith—Freshman Math., pp. 1–13.
4. Schultze and Breckenridge—El. Algebra, pp. 1–25.
5. Smith and Reeve—Essentials of Algebra. Bk. II., pp. 31–34.

C. Subject Matter:

1. Use of formulas (general discussion). Nunn—Ex. in Algebra, Bk. I, pp. 23–32. The following list suggests the scope of the use of formulas:

   a. $V = 4/3\pi r^3$—geometry.

   b. $P = \dfrac{V_1 P_1}{V}$ —physics (Boyle's law).

   c. $S = 1/2gt^2$—law of gravity.

   d. $R = \dfrac{pl}{a}$ —radio.

   e. $C = \dfrac{nE}{nr + r}$ —electricity

   f. $A = P(1 + r)^n$ —compound interest.

   g. $\text{H.P.} = \dfrac{D^2 N}{2.5}$ —auto mechanics.

h. $P=2C-0.01C^2$ —gas engine formula.

i. $r_1=\dfrac{Nr}{1+(N-1)r}$ —statistics.

j. $H=\dfrac{D(h-E)}{d_1-d_2}+h$ —surveying.

2. Construction and interpretation of formulas. (See Mullins, pp. 1–3; Smith and Reeve, Complete Course, pp. 6–7). Examples: (1) Write a formula showing the volume of a cube as a function of the edge. (2) Translate the formula $S=16t^2$ (for falling bodies) into a rule.
3. Evaluation and solving for any letter. The evaluation of such formulas as the following and those just previously listed may necessitate a review of some elementary principles.

   a. $V=e^3$–powers.

   b. $h=\sqrt{a^2+b^2}$ – roots.

   c. $l=a+(n-1)d$—parenthesis.

   d. $C=5/9(F-32)$—If F is 20 a negative number results.

   e. $S=2\pi rh+2\pi r$—factoring.

   f. $\dfrac{1}{n}=\dfrac{1}{a}+\dfrac{1}{b}$ —fractions.

   g. $A=P(1+r)^n$—special products.
4. Interpretation of graphs of formulas.
   Smith and Reeve, Bk. II, pp. 8–16.
   Mullins and Smith, pp. 40–42.
   Christofferson, H. C., THE MATHEMATICS TEACHER, April, 1928.
5. Analyses of formulas as to their description of variation. (See Smith and Reeve, Complete Course, pp. 249–251, Swenson, pp. 278–230.)
   Examples: $C=\pi d$ states that the circumference of a circle varies directly as the diameter.

$A=\pi r^2$ states that the circumference of circle varies directly as the square of its radius.

$\frac{P_1}{P_2}=\frac{V_2}{V_1}$ states that the volume of gas varies inversely as the pressure.

$S=2\pi rh$ states that the lateral area of a cylinder varies jointly as the radius and the height.

III. *Trigonometric Functions*

(If this topic has been well treated in the ninth grade it may be omitted from this course.)

A. Objectives:

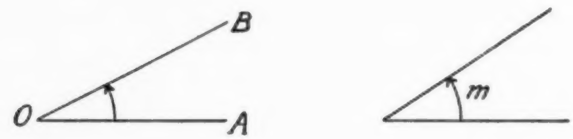
1. To develop an understanding of the difference between direct and indirect measurement with an appreciation of the need for the latter.
2. To develop the following abilities:
   a. To draw to scale.
   b. To use the tangent, sine, and cosine in problems involving right triangles.
   c. To develop the values of functions of special angles and to use complete tables.
   d. To construct the graphs of the sine, cosine, and tangent from 0° to 90°.
   e. To check all solutions roughly.

B. References:

1. Schorling, Clark and Rugg—Modern Mathematics—Briefer Course, pp. 182–196.
2. Smith and Reeve—Essentials of Algebra—Complete Course —pp. (273–290).

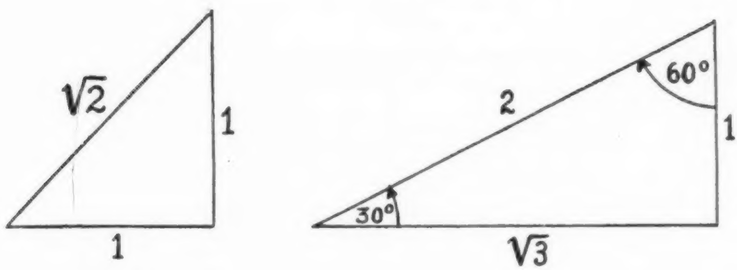
C. Subject Matter:

1. Indirect measurement.
   a. Methods:
      (1) Scale drawing
      (2) Use of similar right triangles and proportion.
   b. Concept of the meaning of "angle of elevation" and "angle of depression."
   c. Notation in naming an angle:



d. Instruments used in measuring angles:
   (1) Compass
   (2) Protractor
   (3) Transit
e. Problems employing scale drawings and proportion.

2. Functions and applications.
   a. Tangent
      (1) A ratio $\left(\frac{\text{side opposite}}{\text{side adjacent}}\right)$ and multiplier.
      (2) Methods of finding tangents:
         a. Graphing
         b. Tables (not over 4-place)
      (3) Problems
   b. Sine
      (1) A ratio $\left(\frac{\text{side opposite}}{\text{hypotenuse}}\right)$ and multiplier.
      (2) Methods of finding sines:
         a. Graphing
         b. Use of tables (not over 4-place)
      (3) Problems
   c. Cosine
      (1) Ratio $\left(\frac{\text{side adjacent}}{\text{hypotenuse}}\right)$ and multiplier.
      (2) Methods of finding cosines
         a. Graphing
         b. Tables (not over 4-place)
      (3) Problems
   d. Problems involving the three trigonometric functions above.
3. Functions of special angles:
   a. 45°
   b. 30° and 60°



c. 90° and 0°

4. Graphs of sine, cosine, and tangent functions from 0° to 90°.

IV. *Elementary Principles of Statistics*

A. Objectives:

1. To develop the ability to interpret statistical graphs.
2. To develop a critical attitude regarding graphs of newspapers, magazines, books, and the like.
3. To understand such terms as *median, mean, mode, quartile, percentile, decile, normal curve.*
4. To know something of the use of graphs in modern business, education, and the like.

B. References:

1. Garrett, Statistics in Psychology and Education.
2. Brinton, Graphical Methods of Representing Facts.
3. Schorling and Reeve, General Mathematics, Book I, Chapter 10.
4. Wentworth and Smith, Commercial Arithmetic.

C. Subject Matter:

1. Graphs
   a. Kinds
      (1) Bar—100% bar graphs
      (2) Circle
      (3) Line
      (4) Formula curves
      (5) Miscellaneous
   b. Point to be carefully noted
      (1) Location of zero point
      (2) Selection of units
      (3) Indication of scale

(4) Table of numbers
(5) Authority for data
(6) Title—clear, brief, accurate, catchy, preferably located at top.
(7) General attractiveness of graph.

c Fallacies of graphs
(1) Optical illusions in circle graphs.
(2) Common fallacies in area and volume pictograms.

d. Examination of published graphs for strong and weak features.

2. Frequency table
   a. Class intervals
   b. Class limits
   c. Practice in making tables
3. Measures of central tendency
   a. Mean—arithmetic average
   b. Median—computation of same
   c. Mode
   d. Advantages and disadvantages of the three above.
4. Quartiles, deciles, percentiles
5. Meaning of normal distribution.
6. Meaning of random sampling.

V. *Linear Functions*

A. Objectives for Linear Functions.
1. To develop the understanding that a graph is the locus of a point which satisfies certain conditions.
2. To recognize the forms in which a linear equation may appear, with the ability to find the slope in each case.
3. To solve equations singly and simultaneously by graphing.

B. References to Linear Functions:

Swenson, High School Mathematics—Chapter VII.

C. Subject Matter:
1. Graphs of linear functions.
   a. Straight line as a locus.
      (1) Have pupils plot points whose sum is always a given value or whose x coordinate exceeds the y coordinate by some number and then see that the result of many plottings gives a line.

b. Slope of line.
  (1) Observe y increase in relation to x increase.
  (2) Define this ratio as the slope.
c. Equation of the straight line.
  (1) Slope intercept $y=mx+b$.
    (a) See what m and b mean by plotting the line.
    (b) $y=mx$, $b=0$.
      Show that this is a special case of (1).

Optional—
  (2) Point slope $(y-y_1)=m(x-x_1)$.

Optional—
  (3) Two point $\dfrac{y-y_1}{y_2-y_1}=\dfrac{x-x_1}{x_2-x_1}$.

  Have pupils find equations of lines with conditions given as in (2) and again as in (3).

  (4) Two intercept $\dfrac{x}{a}+\dfrac{y}{b}=1$.
    (a) Graph this and have pupils see what a and b mean.
    (b) Teach $ax+by=c$ as the general form of a straight line.
    (c) Practice determining the slopes and intercepts of lines.

2. Simultaneous Linear Functions.
  a. Solution by graphing.
    (1) Indeterminate, determinate equations
      (a) Show why one equation in two unknowns is indeterminate.
    (2) Independent equations.
      (b) Show that the slope is different in the equations.
    (3) Dependent equations.
    (4) Inconsistent equations.
      (a) Show that the lines which represent them are parallel because they have the same slope.
      (b) Solution by substitution.
      (c) Elimination by addition or subtraction. (Optional).
      (d) Solution by determinants.
3. Problems—check all solutions.

VI. *Quadratic Functions*

A. Objectives:

1. Same as for linear functions.
2. To graph quadratic functions and recognize equations as being those of a parabola, circle, ellipse, or hyperbola, knowing also that some of these may occur in more than one form.
3. To understand in higher curves the relation between the general slope of the curve, that is, the number of bends, and the degree of the equation.
4. To see how imaginary roots may occur and why they must occur in pairs.
5. To determine the character of the roots of a quadratic by means of the discriminant.

B. References:

Edgerton and Carpenter "Advanced Algebra."
Chapter XVII.
Smith and Reeve "Essentials of Algebra," Book II, Chapter I.

C. Subject Matter:

1. Solution, Methods of:
   a. Graphing.
      Review and extension of the elementary idea of locus as brought out in plane geometry.
      (1) General form $ax^2+bx+c=0$.
         (a) See how the value of the expression changes as x changes.
         (b) Maxima and Minima.
            Teach maximum value as value greater than all values immediately preceding and following; minimum as less than any values preceding or following.
            Show meaning of maxima and minima on the graph.
      (2) Parabola.
         (a) $y=x^2$
         (b) $y^2=x$
         (c) $y=x^2+c$
         (d) $y=ax^2+bx+c$

(3) Circle.

(a) $x^2+y^2=r^2$

Show that the radius of the circle is the square root of r if center is at origin.

(b) Ellipse

(4) Ellipse.

(a) $x^2+4y^2=r^2$.

(b) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

(5) Hyperbola.

(a) $xy=k$

(b) $x^2-y^2=r^2$

(c) $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$.

Have the pupils graph equations (1)-(5), and be able to recognize the shape of each graph from each of the ten equations given.

Show the meaning of the coefficient and a^2 and b^2 in equations of hyperbola and ellipse as being intercepts.

Show how adding a coefficient to one of the terms in the equation of a circle will change it to an ellipse and how changing the sign in the ellipse will change it to an hyperbola.

b. Factoring.

c. Completing the square.

d. Formula.

(1) Theory of quadratics.

(a) Sum of roots $-\dfrac{b}{a}$, product of roots $\dfrac{c}{a}$.

Have pupil discover by the formula what the sum and product is and verify this by equations of which he already knows the roots.

(b) Interpretation of equation.

By discriminant. (Make sure the pupil becomes familiar with these terms.)

$b^2-4ac=0$; roots real, equal, rational.

b^2-4ac, negative; roots imaginary.

b^2-4ac, positive; roots, real and unequal.
If a perfect square; roots equal.
If not a perfect square; roots unequal.

Have pupils see how all these characteristics of the roots are really obtained from the discriminant by practice with equations. Make them reasonable so that knowing them will not be a memory process. Compare the appearance of the graph with the value of the discriminant in various equations.

(c) Imaginary roots.
In quadratic functions show how by changing equations the parabola moves up and the roots become equal and then imaginary.

Real unequal roots.
$x^2-5x+6=0$.

Equal roots; i.e., double root.
$x^2-6x+9=0$.

Imaginary roots.
$x^2+3=0$.

2. Solution of sets of quadratic functions
References; Edgerton and Carpenter
"Second Course in Algebra" Chap. XI
Wells and Hart "Modern Second Course in Algebra" Chapter X.

a. Graphing.

The graphing of a number of sets of quadratic functions is desirable. It is advised that the pupils be taught by diagrams to see the way in which quadratics may have common roots and the number that the various pairs may have in common. It is suggested that Mr. Swenson's Sign Line Method may be taught in connection with these graphs.

The following pairs are suggestions as to ones that may be used.

(1) Parabola and Line

$$x^2-y=5$$
$$x-y=-1$$

(2) Hyperbola and Line

$$2x^2-y^2=4$$
$$3x+5y=4$$

(3) Ellipse and Line

$$\frac{x^2}{4}+\frac{y^2}{9}=1.$$
$$x+y=3$$

(4) Parabola and Circle

$$y+4=x^2$$
$$x^2+y^2=10$$

(5) Parabola and Hyperbola

$$y^2-x-8=0$$
$$x^2-y^2=4$$

(6) Circle and Hyperbola

$$x^2+y^2=25$$
$$x^2-y^2=9$$

(7) Ellipse and Circle

$$x^2+4y^2=16$$
$$x^2+y^2=9$$

Optional:

b. Substitution—as in such equations as:

$$2x+y=7$$
$$x^2+xy=12$$

Optional:

c. Elimination by Addition or Subtraction as in

$$y^2-3x^2=-3$$
$$4y^2+x^2=40$$

VII. *Higher Functions* (*optional*)

1. Graphing as in
   a. $x^3=y$, $y^3=x$
   b. $x^3-4x=0$
   c. $y=x^4+4x+5$.

It is quite probable that the pupil will graph none of these. However, it is advised that they be taught these things:

(1) The general character of higher degree equations; that is, how many roots each must have and so how many bends.

(2) The triple roots as in case of $x^3=y$, $y^3=x$.

(3) How double roots may occur in either third or fourth degree equations.

(4) That imaginary roots must occur in pairs.

(5) That any equation of higher degree than the second must have some—at least one—real root.

VIII. *Exponents, Radicals, and Logarithms*

A. OBJECTIVES:

1. Pupils should understand the meaning of the different kinds of exponents and radicals and should acquire skill in changing them to the most useful forms.
2. Logarithms should be known as exponents.

B. SUBJECT MATTER:

1. Exponents.
   a. Laws.
      (1) Multiplication $a^m a^n = a^{m+n}$.
      (2) Division $\dfrac{a^m}{a^n} = a^{m-n}$.
      (3) Power of a power $(a^m)^n = a^{mn}$.
      (4) Power of a product $(a^m a^n)^r = a^{r(m+n)}$.
   b. Meaning of zero as an exponent: $a^0 = 1$.
   c. Meaning of negative exponents: $a^{-n} = \dfrac{1}{a^n}$.
      Nunn (Successive division).
   d. Meaning of fractional exponents:
      $$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m.$$
   e. Standard number:
      5140 an ordinary number $= 5.14 \times 10^3$ standard no. (Nunn page 175.)
   f. Historic note:
      (1) In 14th century Nicole Oresme conceived of the fractional exponent.
      (2) John Wallis 1659 advanced the use of fractional exponents to negative numbers.
2. Radicals.
   a. Approach by finding areas of square lots.
   b. Change of index

$$\sqrt[3]{5^2}=\sqrt[6]{5^4}$$

c. Emphasize that the radical form is another form of a fractional exponent.
d. Imaginary numbers; definition of "*i*."
e. Drill on manipulation.
  (1) Addition of radicals.
  (2) Subtraction of radicals.
  (3) Multiplication of radicals.
  (4) Division of radicals.
  (5) Rationalizing denominators.
  (6) Finding roots.
    (a) Use of tables.
    (b) $\sqrt{a^2+r}=a+\dfrac{r}{2a}$ Approx.
    (c) $\sqrt[3]{a^3+r}=a+\dfrac{r}{3a^2}$ Approx.
f. Radical Equations—limited to simple forms.
  (1) Extraneous roots, $x+2\sqrt{x-1}=4$.

3. Logarithms.
  a. Labor-saving device.
    (1) Two methods of multiplying.
    (2) Powers of ten.
  b. Logarithms, as exponents.
  c. Characteristic, integral part.
  d. Mantissa, decimal part.
  e. Reading and using tables.
  f. Logarithms.
    (1) Product.
    (2) Power.
    (3) Quotient.
  g. Position of the decimal point.
  h. Interpolation—proportion.
  i. Extraction of roots by logarithms.
  j. Exponential equations—logarithmic equations, logarithmic curve.
  k. Logarithms applied to compound interest and trigonometry.
  l. Historic material.

(1) John Napier 1550–1607 Scottish Baron first used logarithms to simplify arithmetic.
(2) Henry Briggs 1561–30 Prof. in London first constructed tables to the base 10.

4. Slide Rule.
   a. Explanation of the principle of the slide rule.
   b. Squares and square roots.
   c. Multiplication.
      (1) Relate geometrically to logarithms.
   d. Division.
   e. Proportion.
   f. Reciprocals.
   g. Cube roots.
   h. Sines.
   i. Tangents.
   j. Drill as a checking device.

IX. *Series*

The aim of this topic is to develop and understand the series as a foundation for development of further related work.

A. Objectives:

1. To understand the nature of an arithmetic and geometric progression.
2. To develop ability to derive and use the formulas for l and s in an arithmetic and geometric progression.
3. To know that an infinite decreasing geometric series illustrates the idea of a limit.

B. References:

1. Nunn—The Teaching of Algebra—Chs. XIX and XXII.
2. Newell and Harper—High School Algebra Complete.
3. Breslich—Third Year Mathematics—Ch. X.
4. Smith and Reeve—Essentials of Algebra—Complete.

C. Outline of Material.

1. General meaning of series.
   a. Law of Series.
   b. Meaning of expressions.
      (1) Terms.
      (2) Finite.

(3) Infinite.
(4) Sequence.

2. Arithmetic series or progressions.
   a. Meaning of terms.
      (1) Progression.
      (2) Regression.
      (3) Common difference.
      (4) Elements.
   b. Developing of formulas.
      (1) $l=a+(n-1)d$
      (2) $S=n/2\ (a+l)$
      (3) $S=n/2\ [2a+(n-1)d.]$
   c. Arithmetic mean.
      (1) $A=\frac{a+l}{2}.$
3. Geometric series or progressions.
   a. Meaning of terms.
      (1) Ratio.
      (2) Rising or ascending progressions.
      (3) Falling or descending progressions.
   b. Developing of formulas.
      (1) $l=ar^{n-1}$
      (2) $S=\frac{ar^n-a}{r-1}.$
      (3) $S=\frac{lr-a}{r-1}.$
      (4) $S=\frac{a}{1-r}$
   c. Geometric mean.
      (1) $G=\pm\sqrt{ab}$
4. Infinite series (Optional).
   a. Convergent series.
   b. Divergent series.
   c. Oscillating series.
   d. Repeating decimal.

X. *Binomial Theorem*

Several modern texts give little or no treatment to this subject. Those that treat it usually make it a part of series.

A. Objectives:

1. To understand how to write the letters and exponents of powers by inspection; that is, by rule.
2. To know how to use the binomial formula in making expansions.
3. To be able to write the first few terms of the expression $(a+b)^n$, correctly.
4. To know that the binomial formula is the mathematical basis for compound interest.
5. To use the binomial theorem in problems involving compound interest.

B. References:

1. Essentials of Alg., Bk. II—Smith and Reeve, p. 463–70.
2. Freshman Mathematics—Mullins and Smith, p. 56 (problems).
3. Second Course in Alg.—Nyberg, p. 215 (factorial notation).
4. Second Course in Alg.—Sykes Comstock, p. 318 (problems).
5. Algebra, Bk. II.—Longley and Marsh, p. 40 (problems for expression) p. 413–414 (problems for required terms).

C. Subject Matter Outlined

1. Introduction.
   a. Actual multiplication of $(a+b)^n$.
      When $n=3, 4, 5, 6$.
      (1) Meaning of expansion.
      (2) Example of an identity.
   b. Pascal's triangle—(This can be used as an interesting side light or for actual work in coefficients).
2. Facts learned from Introduction (a) and (b).
   a. List all facts observed from 1-(a) and sum up into rule for expanding $(a+b)^n$.
3. Definition.
   a. By formula from rule.

$$(a+b)^n=a^n+na^{n-1}b+n\frac{(n-1)}{2}a^{n-2}b^2+\cdots\cdot.$$

b. Explain factorial notation.

4. Proof.
   a. For small values of n; as when $n=3$.
      (1) Multiply out.
   b. For larger values of n—(This is optional and may be used for particularly brilliant children.)
      (1) By letting $n=?$ (by mathematical induction).
5. Problems such as $(a+6)$ or $(x+y)$ etc.
   a. Explain—sign in problems such as $(x-y)^n$ etc.
6. Formula for kth term (Strictly optional).
7. Application to compound interest.
   a. Explanation of compound interest.
   b. Formula (General).
      (1) To be derived from formula for \$1 as principal.
      (2) $A=P(1+r)^n$.
   c. Problems.
      (1) Such as: Find the compound amount (Int. compounded annually) of \$1500 for 5 yrs. at 4%.
   d. Formula for interest
      (1) $i=P(1+r)^n-p$.
8. Apply logarithms to Interest.

Trigonometry

A. OBJECTIVES:

1. To learn the fundamental trigonometric functions.
2. To use these functions in determining distances by indirect measurement.

B. REFERENCES:

1. "Essentials of Plane Trigonometry," Smith-Reeve-Morss.
2. "Plane Trigonometry," Robbins.
3. "Plane Trigonometry and Applications," Wilczynski-Slaught.
4. "Plane Trigonometry," Granville.

C. SUBJECT MATTER:

1. Trigonometric Functions of any angle, including the study of the unit circle.
   a. Positive and negative angles.
   b. Signs of the functions.
   c. Geometric method.

d. Reduction of any angle to an acute angle.
e. Functions of negative angles.

2. Circular measure of angles.
a. $1^\circ = 180$ radian.
b. Reduction of radians to degrees.
c. Reduction of degrees to radians.

3. Graphs of the functions
a. Sine curve—cosine curve.
b. Tangent curve—cotangent curve.
c. Secant curve—cosecant curve.

4. Relation between functions.
a. $\sin x = \dfrac{1}{\csc x}$.
b. $\cos x = \dfrac{1}{\sec x}$.
c. $\tan x = \dfrac{1}{\cot x}$.
d. $\tan x = \dfrac{\sin x}{\cos x}$.
e. $\sin^2 x + \cos^2 x = 1$.
f. $\sec^2 x = 1 + \tan^2 x$
g. $\csc^2 x = 1 + \cot^2 x$.

5. Functions of the sum and difference of two angles.
a. $\sin (A+B) = \sin A \cos B + \cos A \sin B$.
b. $\cos (A+B) = \cos A \cos B - \sin A \sin B$.
c. $\sin (A-B = \sin A \cos B - \cos A \sin B$.
d. $\cos (A-B) = \cos A \cos B + \sin A \sin B$.
e. $\tan (A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.
f. $\tan (A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

6. Functions of twice an angle.
a. $\sin 2A = 2 \sin A \cos A$.
b. $\cos 2A = \cos^2 A - \sin^2 A$.
$= 1 - 2 \sin^2 A$.
$= 2 \cos^2 - 1$.

c. $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$.

7. Functions of angles in terms of functions of half the angle.

a. $\sin \dfrac{A}{2} = \pm \sqrt{\dfrac{1 - \cos A}{2}}$.

b. $\cos \dfrac{A}{2} = \pm \sqrt{\dfrac{1 + \cos A}{2}}$.

c. $\tan \dfrac{A}{2} = \pm \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$.

8. Sums and differences of functions.

a. $\sin C + \sin D = 2 \sin \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$.

b. $\sin C - \sin D = 2 \cos \dfrac{C+D}{2} \sin \dfrac{C-D}{2}$.

c. $\cos C + \cos D = 2 \cos \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$

d. $\cos C - \cos D = 2 \sin \dfrac{C+D}{2} \sin \dfrac{C-D}{2}$.

9. Inverse trigonometric functions.
10. Trigonometric equations (Limited number).
11. Solutions of oblique triangles.
    a. Two angles and any side.

    b. Law of Sines $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

    c. Three sides.
    d. Law of cosines $a^2 = b^2 + c^2 - 2bc \cos A$.
    e. Two sides and the included angle.

    f. Law of tangents $\tan \dfrac{A-B}{2} = \dfrac{a-b}{a+b} \cot \dfrac{C}{2}$.

g. Two sides and the angle opposite of one of them (the ambiguous cases).

h. Application of logarithms to oblique triangles.

12. Area of a triangle.

a. Heron's formula $A=\sqrt{s(s-a)\ (s-b)\ s-c)}$
(Algebra)

$$A=\frac{b^2 \sin A \sin C}{2 \sin B}.$$

(Trigonometry)

13. Checks for work.

a. Slide rule.

b. Mollvide's formula.

$$\frac{a-b}{c}=\frac{\sin 1/2\ (A-B)}{\cos 1/2C}.$$

14. Field Trigonometry.

a. Plane Table and Accessories.

(1) Mapping an area by method of radiation-scale drawing.

(2) Mapping an area by method of intersection. Finding the total area of given field by the trigonometric solution of triangles.

b. Transit or Sextant.

(1) Measurement of horizontal angles of vertical angles.

(a) Manipulation and use of vernier scale.

(b) Solution of simple practical problems in heights.

(2) Altitude of the sun.

(3) Profile leveling with transit or level.

(a) Construction of a simple profile map with gradient and drain.

(4) Danger angle with the sextant.

(5) Stadia surveying.

c. References.

(1) "Essentials of Trigonometry," Smith-Reeve-Morss.

(2) "Third Year Book"—The National Council of Teachers of Mathematics.

(3) "Elementary Survey"—Breed and Hosmer.

(4) "Introduction to Astronomy"—Moulton.

(5) "Sextant, How to Use it"—Shuster-Bakst

Note: Many of the practical field instruments used in field mathematics for indirect measurement can be easily constructed and will serve the purpose just as well as the more expensive machine made instruments. Some of these instruments are: the Clinometer, the hypsometer, the plane table, the sextant stadia tube, etc.

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—THE EDITOR

A New Idea for Review Work

By WALTER H. BLAISDELL
Nathan Hale Junior High School, New Britain, Conn.

IF YOU, AS A TEACHER of mathematics, lack the time for needed review work in your classes or if your pupils tend to loiter in the corridors or to take more time than is needed to settle themselves for the work of the period, you may be able to find a partial solution of your problems below.

Fundamentally the plan was originally developed to utilize for review purposes the two or three excess minutes that often occur between periods. The evolution of this plan resulted in unexpected disciplinary developments that in some cases tended to overshadow the reviewing aspect of the original motive.

Of the two points of view, those of review and of discipline, it is proposed to consider the latter first. To do this, it is necessary to mention the outline of the mechanical process, which is described later at greater length. Briefly, a small piece of work is placed on the blackboard before the class in question is due to appear. Paper for the pupils' immediate needs is made readily accessible and other arrangements are made to facilitate the speedy completion of the work by each pupil. The quick response of the pupil in his readiness to coöperate is responsible for the disciplinary advantages to which reference is made in the paragraph above:

1. If the piece of work mentioned above is ALWAYS on the board before the first member of the class arrives, a noticeable result may be observed within a very few days, i.e., the class members arrive earlier, oftentimes before the previous class has entirely departed. The reason for this seems to be due to a spirit of competition among class members, a sense of missing something on the part of the pupil if he is not there, and an eagerness to react to a challenge of knowledge, intelligence, and ability to think quickly. This effect is noticeable in groups of high and low intelligence alike, regardless of their other differences.

2. The thoughts of the pupil are automatically turned into channels appropriate to the period. The reason for this need not be enlarged upon but generally may be attributed to the several means of interest stimulation among class members.

3. When this given piece of work is completed, the average pupil proceeds to whatever other work he may have at hand. Thus the lapses so often occurring at the beginning of a class period tend to disappear. Incidentally, such a procedure proves an excellent sedative for the physically active members of a group, especially among the more immature pupils who have not learned to be economical with their time.

4. The pupil's errors are forcibly brought home to him. He studies his fundamental rules and their applications more thoroughly. He can compare the results of his own efforts with those of his associates, especially as these results are recorded on class record cards and placed on the class bulletin boards.

The mechanical details of this procedure as practiced by the writer are given below. Other teachers report that modifications of this operation work effectively in other classes such as History, English and General Science.

Before a class is due to arrive, the instructor copies a previously chosen question or example on the blackboard. The nature of that question is described later. As the first pupil appears, he obtains the number of sheets needed by his class from a supply previously prepared. He places the approximate number of sheets required on the front desk of each row, from which they are distributed as other pupils arrive. As the work of the individual pupil is completed, it is collected, usually by the one that first finished the example. Another pupil makes a copy of the original question marked with the date when given on a sheet used for that purpose and which is tucked behind the record card of that class when not in use. The writer uses this record of daily miniature tests and their results as a basis for extra credit, also at times for corrective work. The reader will observe that the mechanics of the operation are handled entirely by the pupils, except for the writing of the original question and its answer. It is a matter of fact that by the time the opening bell of the period has rung, nine times out of ten the review work is finished and the class is ready for the work of the period. The corrector of review examples may, if desired, postpone the correcting and checking of the answers until a time when he is at leisure.

The nature of the daily questions or tests requires some consideration. Generally, three things should be observed in their choice: (1) The question must deal with fundamental subject matter. (2) The question must be relatively simple, involving no ambiguity of question or answer. (3) The question should be of such a nature that its answer is informative as to the actual amount of knowledge

concerning the subject matter of the question that the pupil has acquired.

The reader will observe the justification of these qualifications when he considers that the answers are to be checked by a pupil, whose class work must not be encroached upon more than is necessary. A simple, short answer is much to be desired. Multiple answers are hard to correct and if a certain amount of credit is to be given for work that is partially correct, the work of the corrector is greatly increased, and the results on the record card are not a true indication of the pupil's ability. Then too, if a pupil knows beforehand that he has a fair chance of answering correctly the question to be given, he will show a much greater degree of interest than he would if he knew that he would be expected to solve a complex problem or answer a puzzle question.

The possible uses to which the data acquired from these daily tests may be put are several: (1) They serve as an excellent additional check on the work of the individual pupil. (2) They show to what degree he is interested in his subject, inasmuch as this activity is not required, coming as it does before the work of the period actually begins. (3) In view of the fact that the time for each question is limited, these data measure the pupil's ability to think quickly, also his degree of pertinent fact retention.

All review work may be divided into three distinct classes: (1) A review of fundamentals from the beginning. (2) A review of the work of the immediate past. (3) A review to prepare a setting for coming subject matter. In the first case, it is advisable at times to call to the attention of the pupil work that may have been satisfactorily completed several months before and to which reference has not recently been made. In the second case, any teacher can call to mind instances in which a large portion of a period has been spent in explaining a new principle, without afterward knowing how much of that explanation the pupil can recall several days later. In the third case, the instructor often finds it desirable to call to the attention of the pupil certain phases of previously studied subject matter because of their relation to new work that will arise in the immediate future. The writer has experienced some advantage in reviewing arithmetical fractions in preparation for algebraic ones. In these three classes of review material, the daily review tests accomplish the intended pur-

pose of presenting to a class special phases of subject matter without disturbing the program of the period.

The advantages of this plan may be summarized as follows:

1. Pupils come into class earlier, helping to clear the corridors.
2. Each pupil has an incentive to utilize his time from the moment that he enters the room.
3. The pupil is induced to break his thought connections of the previous period and to pick up new trends of thought appropriate to the coming period.
4. The teacher has a record of many questions and answers, which record he may use to divers advantages.
5. Last but not least, a detailed organization of the responsible pupils of the class eliminates all but a minimum of effort on the part of the teacher.

Once more we wish to remind our readers that if the National Council of Teachers of Mathematics is to really do the work that needs to be done to improve the teaching of mathematics in the United States of America, we shall have to get more of the rank and file—the classroom teachers—interested in what we are trying to do.

Every teacher of mathematics in the elementary and secondary schools should be a member of the National Council and should read THE MATHEMATICS TEACHER and the National Council Yearbooks if they are to keep up to date in their work. The only way in which the Council can grow is for everyone who has any interest in the Council to lend a helping hand.

The Editorial Committee is constantly being told about large areas of people where the publications of the Council are rarely taken or read. If this is true, it is a reflection on the rest of us. Can't we all work just a little harder for the subject in which we should be the most interested?

THE MATHEMATICS TEACHER will be glad to send advertising material to anyone who requests it to be used at teachers' meetings of any kind or will send out such material to anyone who may be designated by any friends of the Council. Now is the time to start an active fall campaign for new members.—THE EDITOR

Methods of Arithmetic Problem Solving

By Paul R. Hanna
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I. *Purpose*

The purpose of this study is to compare experimentally the merits and defects of three methods of arithmetic problem solving. This experiment attempts to measure the gains in ability to solve problems which follows an extended period of drill on the three experimental factors or methods* of problem solving.

II. *Sources of Data and Techniques Used*

Three Experimental Factors. The three methods of problem solving used in this experiment were called the dependencies (graphic or diagrammatical), the conventional-formula (four step), and the individual (absence of any formal method). The first of these techniques, or methods, was designated as experimental factor A; the second, experimental factor B; and the third, experimental factor C. An explanation with illustration of these three reasoning types is necessary at this point.

Experimental Factor A: Dependencies Method. This method directs the pupil to follow a particular thought pattern in each solution. The pupil determines first what is to be found in the problem. The answer to this *depends* upon certain factors of the problem and each factor is likewise *dependent* upon other factors, and so on, until the pupil has unraveled the essential facts and *dependencies* in the problem. The following problem solution will illustrate:

John had 16 cents. He earned 10 more and then spent 15 cents for ice cream. How much money did John have left?

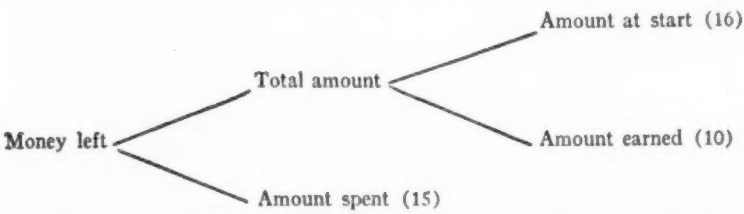
The pupil is directed to think of what is asked for:

I am to find the *money* John has *left*. To find how much *money is left* I would have to know the *total amount* of money he had and the *amount*

* Throughout the experiment the terms, method, analysis, procedure, and technique, are used interchangeably as synonyms.

spent. To find the *total money* he had, I would have to know the *amount* he had *at* the *start* and the *amount* of money he *earned.*

These thought processes can be pictured graphically as follows:



With this logical analysis of the dependent factors in the problem, the pupil then makes the necessary computations.

Experimental Factor B: Conventional-Formula. This method directs the pupil to follow a particular thought pattern in each solution. This pattern has the following four steps:

1. What is asked for in the problem?
2. What is given in the problem?
3. How should these facts be used to secure the answer?
4. What is the answer?

The following problem solution will illustrate these steps:

John had 16 cents. He earned 10 more and then spent 15 cents for ice cream. How much money did John have left?

In order to analyze the problem, the pupil fills in the first two questions of the formula as follows:

1. What is asked for? *Money left.*
2. What facts are given in the problem?
   Amount of money John had (16)
   Amount of money John earned (10)
   Amount of money John spent (15)

These facts being recorded, the pupil now thinks through the steps necessary to combine these facts into an answer. Thus he writes the following as an answer to question three:

3. How should these facts be used to secure an answer:
   (a) $16 + 10$ (b) $a - 15$

He then does the necessary computations for (a) and (b): and has the problem answer which he puts down as answer to question 4.

4. What is the answer:
   11 = amount of money left

Experimental Factor C: Individual Method. In this method the children are allowed to use whatever method of analysis they desire to solve problems. This constituted the method used by the control group in this experiment. The method can be illustrated by the following problem solution:

John had 16 cents. He earned 10 more and then spent 15 cents for ice cream. How much money did John have left?

A child would probably add 16 and 10 getting 26; then subtract 15 and have a remainder of 11 or John would have eleven cents left. If the child found the computation too difficult to do mentally, the work on paper would appear as follows:

$$\begin{array}{r} 16 \\ +\,10 \\ \hline 26 \\ -\,15 \\ \hline 11 \end{array}$$

Any other logical method or short cut which the individual child might select for solution was acceptable in this group. A definite effort was made to prevent the teacher or pupil from inducing the class to follow any uniform pattern in the problem solution. It was the purpose of the experiment to measure the relative worth of a uniform pattern of thought procedure, dependencies method (Method A) and conventional-formula (Method B), with the lack of any uniform formal technique (Method C). This experimental group served as a control and in addition furnished evidence in support of or contradiction to the belief that children do better with definite patterns for problem solution.

Sources of Methods. The *dependencies method* (Method A) is a very new one to arithmetic. Dr. John R. Clark presented it in an article in THE MATHEMATICS TEACHER, April, 1925.

The *conventional-formula method* (Method B) is similar to the conventional technique which is the most often used in textbooks and professional literature. A difference exists between the usual conventional method and the method called conventional-formula in this experiment. The third step of this analysis, namely: What operations are necessary? is generally conceived to require a verbal response of the name of the necessary operations; for instance, the

pupil might answer the third step by stating, "Multiply, then divide." However, in order to make sure that the pupil really arrived at these operations by synthesis of the facts in the problem, a written formula was introduced in this third step. In this experiment the pupil was required to state the facts together with proper operation symbol in a simplified formula as illustrated in the foregoing problem. In so far as there may have been additional difficulty inherent in this type of written response, the conventional method as used in the majority of textbooks differs from the method labeled conventional-formula in this experiment.

The *individual method* (Method C) is undoubtedly the most widely used of all methods today. Although the conventional method is advocated in literature, it is generally the exceptional teacher or child who follows the more formal or definite technique. Evidence is lacking that any children in this experiment had previously had any consistent drill with any method of problem solving.

The Subjects. Approximately 1,000 children participated in this experiment. In the paired groups a total of 477 was used as subjects. Twenty-four classes were involved—twelve seventh grades and twelve fourth grades. These classes were selected from three schools, two public schools and one private experimental school, all of New York City. These three schools contain a reasonable sampling of children from urban life.

The Equating of Groups. A battery of four standardized tests was given to the twenty-four classes of children. This battery of tests served two purposes: first, to equate groups; second, to give initial scores in arithmetic. It consisted of two different arithmetic reasoning tests and two forms of the same intelligence test. The New Stone Test in Arithmetic Reasoning (Form 2) was given as test one. Form A of the Pintner Rapid Survey Test in Intelligence was the second test in the series. The Stanford Achievement Test in Arithmetic Reasoning (Form A, Test 5) was given as the third. The final test was Form B of the Pintner Survey.

The twelve classes in each grade were arbitrarily placed in the three experimental groups, with four classes in each of groups A, B, and C. Care was taken to distribute the classes so that at least two methods were represented in each school. After the scores of the various tests had been weighted, a total initial arithmetic score and a total battery score were computed for each child. For each grade

the usual procedure of group equating was followed. The results of this equating are given in Tables *I* and *II*.

TABLE I

SCORES IN ARITHMETIC PROBLEM SOLVING OF THE 4TH GRADE CHILDREN IN EACH OF THE THREE GROUPS

Groups	No. of Pupils	Mean Score	Standard Deviation
Group A	75	64.95	20.32
Group B	75	64.77	19.94
Group C	75	64.62	20.11

TABLE II

SCORES IN ARITHMETIC PROBLEM SOLVING OF THE 7TH GRADE CHILDREN IN EACH OF THE THREE GROUPS

Groups	No. of Pupils	Mean Score	Standard Deviation
Group A	84	142.2	27.16
Group B	84	141.5	27.10
Group C	84	141.8	27.58

The figures on the total battery (arithmetic plus intelligence) give further evidence of the reliability of the equating.

The Practice Problems. Practice problems were carefully selected and were mimeographed, seven problems to each daily practice sheet. After the statement of Problem 1, a sample solution was given, using the method of reasoning peculiar to each experimental group. Each pupil was given a practice sheet. All read the brief instructions. The teacher read through the first problem with the pupils, being careful to follow through the thought processes in the language suggested in the Instructions to Teachers. The second problem was then read and worked in the same way as Problem 1, etc. This work consumed twenty minutes per day.

Instructions to Teachers. In order to equalize the irrelevant factor of teacher ability, definite teaching directions in mimeograph form were given. An hour conference was held with the teachers of each experimental group separately and an attempt made to instruct them thoroughly in the method.

In addition to these very definite directions, each teacher was given on each practice day a key sheet with all problems worked out for her. Thus, there could be little variation in the method of presenting the material to the pupils.

Final Test. At the end of six weeks the pupils had completed the twenty sheets of practice material, spending approximately ten clock hours on practice exercises. A battery of standardized tests was again administered. This final set was limited to two identical test forms of arithmetic reasoning, Stone and Stanford, used in the initial test. After scoring and weighting on the basis of variability, these final scores were set down opposite the initial scores.

Studying the Data. The technique employed of measuring the reliability of the differences of the means is very common in such an investigation. The total initial arithmetic score was set opposite the total final arithmetic score for each individual and an algebraic subtraction made. The differences or gains for each group of pupils thus found were then treated for the mean gain, the standard deviation of the gains, and the standard deviation of the mean. When these computations had been made for all the ability groups of the fourth and seventh grades, the achievements of the various groups were compared.

III. *Results*

Grade Four. The children of the fourth grade regardless of ability grouping made the greatest gain with the *dependencies method.* The chances are 93 in 100 that the true mean gain of the dependencies method will be greater than the true mean gain of the conventional-formula method, and 82 in 100 that the true mean gain of the dependencies method will be greater than the true mean gain of the individual method. The individual method is slightly superior to the conventional-formula method, the chances in favor of such being 67 in 100.

Consideration was then given to the various ability groups of the fourth grade:

a. Considering only the pupils with *superior* arithmetic reasoning ability in this grade, there is no clear evidence of superiority of any of the methods. The pupils seem to have done almost as well with one method as with another.

b. Considering the *average* ability group, again there is no positive evidence of great superiority; the greatest gain was made by the group using the dependencies method. The D./S.D. diff. of .9 indicates the chances are 83 out of 100 that this gain might be expected in another identical situation.

c. Considering the pupils of *inferior* arithmetic ability, the results

are very positive in favor of the dependencies over both the conventional-formula and the individual methods. The chances are 100 in 100 and 99 in 100 that this dependencies method is better than the conventional-formula method and individual method respectively. There is slight advantage attending the individual over the conventional-formula method.

Grade Seven. Pupils of the seventh grade, irrespective of arithmetic ability, made the greatest gains using the individual and dependencies method. The D./S.D. diff. in favor of the dependencies over the conventional-formula method was 1.80 and in favor of the individual method over the conventional-formula method 2.5. Or the chances are 96 in 100 that the dependencies method and 99 in 100 that the individual method are better than the conventional-formula method.

Consideration was given to the various ability levels of the seventh grade:

a. Considering the pupils of *superior* ability as a group, the results are positive in favor of the superiority of the dependencies and the individual over the conventional-formula. The chances are 100 in 100 that the true mean gains of the dependencies and individual methods are greater than the true mean gain of the conventional-formula method.

b. and c. With the *average* and *inferior* ability groups, those using the individual method consistently made the greatest gains. The difference between the dependencies method and the conventional-formula method was negligible for these two ability groupings.

Ability Levels of Both Grades. When the ability levels of the pupils of both grades are considered, there are significant differences in favor of the dependencies and individual methods over the conventional-formula method when the *superior* pupils alone are concerned. The chances are 98 in 100 that the true mean gains of these two methods are greater than the true mean gain attending the conventional-formula method. There is no difference in the mean gains of the dependencies and individual methods.

For the combined *average* ability groups of both grades, no difference is sufficiently great to indicate much more than a fifty-fifty chance in favor of any of the methods. Results indicate a very

TABLE III

RELIABILITY OF DIFFERENCE OF MEANS OF INITIAL AND FINAL TESTS
ALL PUPILS REGARDLESS OF GRADE OR ABILITY

Method	No. of Pupils	Total Gain	Mean Gain	S.D. Gain	S.D.$_m$	Difference of Means			S. D.$_{diff.}$			$\frac{D^*}{S.D._{diff.}}$			No. of Chances in 100 that†
						A−B	A−C	B−C	A−B	A−C	B−C	A−B	A−C	B−C	
A.........	159	1863	11.7	14.3	1.1										A is better than B=99
B.........	159	1226	7.7	16.4	1.3	4.0	0	−4.0	1.7	1.7	1.8	2.35	0	−2.20	A is better than C=50
C.........	159	1859	11.7	16.5	1.3										C is better than B=99

* These figures are to the nearest .05.

† Garrett, p. 134.

Note: This table should be read as follows: For pupils of seventh and fourth grades combined, the chances are 99 in 100 that the mean gain accompanying *Method A* will be greater than that accompanying *Method B*. The chances are even that, etc.

Method A = Dependencies. Method B = Conventional-formula. Method C = Individual.

slight superiority of the individual method over the two other methods.

For the pupils of *inferior* arithmetic ability, the dependencies method appears superior to either the conventional-formula method or the individual method. The D./S.D. diff. of 2.05 indicates that the chances are 98 in 100 that the true mean gain of the dependencies method is greater than that of the conventional-formula method. The chances are 65 in 100 (only a little more than an even chance) that the true mean gain attending the dependencies method is greater than that of the individual method. The chances are 95 in 100 that the true mean gain of the individual method is greater than the true mean gain of the conventional-formula method.

IV. *Summary*

A final study of all pupils disregarding grade level and ability grouping was made by comparing the mean gains of the three experimental factors. (See table III.) The mean gains of the dependencies method and the individual method were alike, namely, 11.7. The mean gain of conventional-formula method was only 7.7. These results give a D./S.D. diff. of 2.35 in favor of the dependencies method over the conventional-formula method and a D./S.D. diff. of 2.2 in favor of individual over the conventional-formula method. Translated, these results indicate the chances are 99 in 100 that the true mean gains of the dependencies and individual methods are superior to those of the conventional-formula method. No difference was evident between the dependencies and the individual methods.

In conclusion, it is evident that the conventional-formula method gives the least mean gain. The statistical results are sufficient to demonstrate a significant difference in favor of the dependencies and individual methods, with no choice between the two.

Notice to Delinquent Members

Members of the Council whose subscriptions to The Mathematics Teacher expired in May, 1930, have all been sent renewal notices to which some have not responded. In order not to cause a break in their files such members have been sent the October number. Obviously, we could not continue these names on our list unless these subscriptions were renewed. We therefore have been compelled to drop from the list the names of all persons whose subscriptions were not renewed by October 15 or when the November issue went to press.

The Editor

Professional Preparation and Growth in Secondary School Mathematics

By LAURA BLANK
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HOWEVER INADEQUATE one finds his undergraduate preparation in mathematics, once he is settled in the work of teaching secondary school mathematics, there is little excuse for his failure to pursue some very definite plan for professional growth. It is true, one can fulfill the conditions of his contract, he can acquire even a sort of proficiency, as an instructor, drawing solely from the fund of information and training of his college days, coupled with his gradually acquired experience. This same instructor, after he has developed a genuine enthusiasm in his subject, a constant watchfulness for allied interests, associations and contacts, after he has pursued some externally directed or self directed study of some branch of mathematics for several years, he rapidly becomes a very valuable member of a department of mathematics.

Just as a world is a composite of all sorts of individuals, so is a department of mathematics, or any other department, for that matter, of university or high school, composed of various types. Among one's colleagues are those who merely fulfill the demands of their contracts, perhaps scarcely that, then those who make some small attempt at getting on, this meager effort evident in the parasitic desire to catch up the threads of plans and endeavor begun by those more industrious and ambitious, and finally the last group, the minority, those who are students, not necessarily scholars, eager for progress, persistently and consistently working for growth and enlarged understanding.

There are so many modes of supplementing one's preparation, however insufficient. If one is a teacher of junior or of senior high school mathematics, whether he is teaching the following subjects or anticipating teaching them or not, he should be the master of plane trigonometry, Euclidean solid geometry, theory of equations, differential calculus and plane and solid analytic geometry, none of which however

need be studied in a class, under a professor. He should also have a thorough course in physics. Needless to say, such a program, if not completed, at least in part, in one's undergraduate student days, can not be thoroughly mastered in a year or in several years along with one's regular professional duties. Yet the program is well worth achieving, since it is invaluable in one's interpretation of his more elementary high school courses; it is even essential so as to avoid gross blunders, and should therefore constitute a minimum attainment.

There are several other sources of self training and growth. The history of mathematics, of which there are a number of authoritative works, by no means difficult reading for a teacher of mathematics, throws much light upon the growth of the various branches of the science, shows much of the human side of this very abstract type of human thinking and explains much of the gradual civilization of the human race. This history is fascinating, a source of understanding and interpretation if read and reread at intervals from year to year. The great figures in mathematics, many of them intensely human, clothe the science in flesh and blood, for our youth. A familiarity with the anecdotes associated with these men, with the account of the development of mathematics out of very genuine human every-day needs, and with the spread of mathematics over the world vitalizes the subject, making more of it than mere abstractions, ready made. After all, it will possibly be granted that the gradual development by the human race of a flexible system of numbers so that it, in time, could count has as largely influenced civilization as the invention of the alphabet.

Another source of professional growth, often less authentic, and less interesting, ideas still in the flux, but well worth knowing, is the group of current professional journals. There are a number of them in the secondary and college spheres voicing the research and convictions of scholars, students, and others, in the fields of pure mathematics, applied mathematics, pedagogic mathematics, scientific mathematics and mathematical science. One can become acquainted with some of the important contemporaneous students of mathematics only through the journals. Read a scholar's or student's very superior textbooks, if he writes any, teach them for years, value them as unusual works. If they are works on mathematics, they are likely impersonal and abstract to the extent that one does not know the

author or appreciate him excepting as a teacher and textbook writer. Read his essays and articles in the journals and one may learn to know the man as a very human creature with the traits of a human and sympathetic and understanding guide for youth. Superior as are textbooks of David Eugene Smith, one does not know the man, though one has taught his texts for years. Read his contributions to the journals and one finds him at once to be perhaps our greatest contemporaneous student of art in the realm of mathematics. His aesthetic interpretations, at the same time rigorously scholarly and mathematical, flood the subject with a light, a charm, a fascination, yet such an appreciation as can be felt and understood by ordinary folk, such as we who are, not necessarily artists. Thus we can realize to some small degree the delight of such great souls as Leonardo da Vinci, artist and scientist, the artist interpreting scientific interests, the scientist elaborating upon, explaining aesthetic enjoyment.

In the contributions to the journals one rarely feels the impress of the textbook publishing house. The author writes usually unhampered by any motive or influence of expediency. He writes, as a rule, with no idea of remuneration, with merely that of helping a colleague, of exchanging investigations or experiences, of seeing a subject from a slightly different personal angle, perhaps unique to the writer. Perhaps his clever reader will use his idea to personal gain. Even so. He writes as a scholar, claiming no prior rights. His motive is professional, social. It is true, knowing textbooks, one occasionally reads between the lines in a journal, recognizing a motive on the part of the author, other than those suggested. Yet this is exceptional. Then the journals are well worth one's study.

Finally, as a source of personal enlightenment there are the current and recent textbooks themselves. One cannot teach his subject intelligently and efficiently if he does not know well those texts which his own pupils have studied. Nor can he teach with acute and discerning understanding a subject which will serve as a foundation, perhaps serve to be inspirational, exploratory, diagnostic, unless he knows well the type of course and textbook to be studied subsequently by his pupils. Should he be teaching trigonometry to a class composed largely of prospective engineers, his efficiency will be much increased if he endeavors constantly to learn the problems and practices and textbooks of present-day engineers and schools of engineering. Perhaps his course is one in the so-called general mathe-

matics, a junior high school subject. One questions the possibility of its being inspirational if the instructor does not know well the larger field and subsequent texts for which he is claiming to provide and incite inspiration and enthusiasm. Parenthetically speaking, there seems to be among educators, an erroneous theory that a teacher of adolescents who has taught arithmetic and is able to read, with some study of it, one junior high school textbook in mathematics is thereby qualified to teach mathematics, not purely arithmetic, to pupils of the junior high school grade. If junior high school mathematics, which is different from arithmetic, or if general mathematics in the junior high school, is ever to succeed properly this will come about when the instructors of these subjects are as thoroughly prepared, as completely masters of mathematics as are some of the instructors of mathematics in the senior high schools. Hence such instructors should be students of all of the senior high school texts. The instructor of mathematics should know not only the texts of his school system, but all available texts. One will be astonished at the number of textbooks, having some bearing upon his work to which he can have access if he wishes it. There are all sorts of texts in different sorts of libraries. One's colleagues have texts different from one's own. Instructors have been known to spend some four weeks with a class in a university graduate school studying and justifying one textbook. Such bigotry is unpardonable, though it is possible even there to glean some new ideas and values, not however commensurate with the time and money expended.

One acquires many devices of technique and method from textbooks never used in his own classroom. He becomes familiar with the broad extremes within which textbooks lie, extremes as to subject matter, examples, illustrations, theory, logical and psychological structure, abstract and concrete applications, etc. Comparing the current textbooks with those published recently and formerly, he learns the trends and tendencies in his subject. With some further investigation and study he may find the causes which function to bring about such tendencies. In this manner he may learn to forecast, to some extent, the future direction or course of some branch of his subject.

In the study of textbooks, the survey may be somewhat superficial, not intensive and analytic in every instance. Needless to say, any study of a textbook which does not involve a daily classroom use of it, over a period of time, can not fail to be somewhat superficial.

The purpose in our textbook interest here is not that of study for adoption or rejection as far as classroom use is concerned, but personal interest, understanding and appreciation. It is noteworthy that one's very careful and painstaking analysis of a text prior to its adoption leads to an estimate of it often very radically changed after he has used the book in class for a semester or a year or two.

One can, by constant endeavor, grow in worth not alone to his own department, but by cooperation become more valuable to other departments of an institution. If mathematics in any school or university is to become the language and tool of many of the sciences, it may be possible to bring about such cooperation between the departments of science and mathematics that courses, and, if possible, topics in two or several departments can be given contemporaneously or nearly so. Thus each subject may serve to reënforce the other, interpret it and coördinate it. If groups of the faculty from the physics, general science, chemistry, and economics departments and mathematics department should meet, from time to time, the former telling the latter in what way their subjects might be of service to the sciences, as taught now in this particular institution, there would be no need for artificially coördinated mathematics. For example, it might be possible to teach Boyle's Law in general science or physics or chemistry, or the law of the lever in general science or physics, immediately after teaching ratio and proportion in mathematics. It is possible that the courses of two departments might be so constructed that the law of the falling body in the experimental science department might be studied just after the same pupils have studied arithmetical and geometric progressions in intermediate algebra. Pupils comment with happy appreciation and understanding when such juxtaposition of subject matter works out quite by accident in two departments. It is possible, perhaps, to study similar triangles and the parabola in mathematics just prior to the study of reflection of light in physics. Many of the topics of algebra may be rearranged and put into an organization, not inconsistent with the theory of the text being used in the course at a particular time, so as to adjust at least some of them to the needs of another department. The study of graphs may be entered into, in the department of mathematics, so as to be of use when problems of physics or general science are later exemplified or reënforced by their application. The organization of algebra and general mathematics textbooks along such very different

lines and embodying such very different theory, if one may judge by the authors' justification of the organization, convinces one that a reorganization motivated by very genuine coöperation within any one system, high school or university, is likely feasible. However, this sort of coöperation or coördination requires time effort, understanding and intelligence. It can not be portrayed complete in any textbook, though textbooks are fertile in suggestion as to cross contacts and points of collaboration.

In such a relatively unimportant matter as the correcting of papers, for instance, in geometry, it is possible for the department of mathematics to agree with the department of English and other departments upon a uniform notation of symbols to indicate errors of the same sort or suggestions for pupil corrections. Thus, pupils in the same institution need not learn two such sets of symbols. If an institution has a department of printing it may be possible to have such a set of symbols put into printed form on a card or on looseleaf notebook paper of standard size so that each pupil may have a copy for reference.

In the matter of notebooks, it is just possible for the various departments to come to agreement upon a uniform notebook of standard type, carrying ruled paper for English and some of the other departments and unruled and coördinate paper for the mathematics and other science departments. Such a policy is generally appreciated by the conscientious student who carries much material home for preparation there.

There are, then, so many ways, of large and small influence, by which one may grow in his position. A class was studying arithmetical and geometric progressions recently. With no warning came the question, "Are there other sorts of series besides arithmetic and geometric?" The instructor who can answer such a question at once inspires confidence in his class, often creates greater interest in his subject. A class was studying graphs involving two variables. This same class, a short time before, had been studying the algebraic solution of linear systems in three variables. One bright lad associating the two solutions or interpretations of the same hypothesis, asked spontaneously, "$2x + 3y = 7$ being a straight line, what would the graphic solution of $2x - y + z = 5$ be, and how would one work it out?" It takes but a few minutes to illustrate, with rulers, the three orthogonal axes, x, y, and z of solid analytics, and to explain the

locus of such an equation. This same class was studying conic sections. In plotting the equation $xy = -4$, a pupil, having transformed the equation to $x = -4/y$, chanced to substitute zero for y. He came upon a situation new to him. What next? Could any artificial situation be created to elicit the interest and curiosity aroused, thus providing an ideal opening for the introduction of the concept, infinity, and the problem of the possible division of a finite number by zero?

Two successive years, now, young women of classes in fourth year high school mathematics, superior pupils, have asked what might be done with mathematics after college. They were not interested in teaching; nor did they wish to pursue such engineering courses as are open to women. They were not interested in research in mathematics or in allied mathematical sciences. Surely the instructor in mathematics should be intelligent enough to answer such pupils, thus pointing out to them possible professional work for which they may show peculiar and unique qualification.

The presidents of progressive universities and colleges are motivated in appointments and promotions on their faculties by the theory that if one is once a student he should always be a student. Should not all teachers be students of human affairs generally, but particularly of those affairs which touch upon their unique subjects of pursuit and interest?

"From the language of statistics there is no escape if we wish to go beyond the limits of personal opinion and individual bias. Worthwhile evaluations in higher education will continue to be as rare as they now unhappily are until the rank and file of college and university teachers become able to think in more exact quantitative terms than they are yet accustomed to."

Introduction to Measurement in Higher Education by Ben Wood. World Book Co., 1927, p. 4.

Cardan

1501-1576

CARDAN AS HE is commonly referred to by English writers is said to have been born at Pavia in 1501 and to have died at Rome on September 21, 1576. His name appears as Girolanio Cardano, Hieronymus Cardanus, and Jerome Cardan. He was "the first of the two prime movers in the solution of the cubic." He was the son of a jurist, Facio Cardano (1444-1524), who was professor of jurisprudence and medicine in Milan and who edited Peckham's *Perspectiva Communis.*[1]

Cardan's life exhibited many contrasts. He delved into astrology while at the same time he was a serious student of philosophy. He is said to have been a gambler and yet a first class student of algebra. His statements were very unreliable while at the same time he was a physicist whose habits and observations could be relied upon. Cardan was a physician and yet the father and defender of a murderer. At one time he was a professor in the University of Bologna and at another time an inmate of an almshouse.

Cardan was responsible for *Ars Magna,* the first great Latin treatise devoted solely to algebra which appeared at Nürnberg in 1545. This treatise set forth the theory of algebraic equations so far as it was known at that time. In it he included the solution of the cubic which he is said to have secured from Tartaglia in spite of a pledge of secrecy and also a solution of biquadratic equation which had been previously discovered by his pupil Ferrari. In spite of his trickery he was a man of great learning who proved himself capable in many different fields.

[1] For a more complete history see Smith, David Eugene, History of Mathematics 1: 295-297, Ginn and Company.

On To Detroit!

The annual meeting of the National Council of Teachers of Mathematics will be held in Detroit, Michigan, on Friday and Saturday, February 20 and 21. National Council headquarters will be at the Detroit-Leland Hotel. Send in your room reservations at once and watch the December issue of THE MATHEMATICS TEACHER for the tentative program.